Problem Set 6 Solution

1) Pg 301, #3

\[ Q = 10K^{0.25}L^{0.25} \]

a) \[ I = wL + rK + \lambda [Q - 10K^{0.25}L^{0.25}] \]

1) \[ \frac{\partial I}{\partial L} = 0 \Rightarrow w = 10^{0.25}K^{0.25}L^{-0.75} \]

2) \[ \frac{\partial I}{\partial K} = 0 \Rightarrow r = 10^{0.25}K^{-0.75}L^{0.25} \]

3) \[ \frac{\partial I}{\partial \lambda} = 0 \Rightarrow Q = 10K^{0.25}L^{0.25} \]

1) \[ \Rightarrow \frac{w}{r} = \frac{K}{L} \Rightarrow K = \frac{w}{r}L \]

3) \[ Q = 10 (\frac{w}{r})^{\frac{1}{2}}L^{\frac{1}{2}} \Rightarrow L^{\frac{1}{2}} = \frac{Q^{\frac{1}{2}}}{10 (\frac{w}{r})^{\frac{1}{2}}} \]

\[ \Rightarrow L^* = \frac{Q^*}{100 (\frac{w}{r})^{\frac{1}{2}}} \Rightarrow K^* = \frac{Q^*}{100 (\frac{w}{r})^{\frac{1}{2}}} \]

b) \[ LRATC = wL^* + rK^* = \frac{Q^*}{100} + \frac{Q^*}{100} \frac{w}{r}r = \frac{Q^*}{50} \frac{w}{r}r \]

c) \[ LRC = LRATC \frac{Q}{Q^*} = \frac{Q}{50} \frac{w}{r}r \]

\[ LRMC = \frac{\partial LRATC}{\partial Q} = \frac{Q}{50} \frac{w}{r}r \]

d) Both LRAC and LRMC increase as Q increases since the production function has decreasing returns to scale (since it's Cobb-Douglas w/ \( \alpha + \beta < 1 \))

e) \[ I = \frac{w}{d^5}L^{0.75}K^{-0.75} = \frac{w}{d^5} \left( \frac{Q^*}{100} \frac{10}{r} \right)^{0.75} \left( \frac{Q^*}{100} \frac{w}{r} \right)^{-0.75} \]
e) continued: \[ A = \frac{w}{\delta} \frac{Q}{10} \frac{\delta}{V_W} = \frac{Q}{25} \frac{\delta}{V_W} = LRMC. \]
The same as LMC!

2) \[ TC = 10Q^3 - 60Q^2 + 100Q \]
   a) The long run supply curve is flat, i.e., the inverse supply curve is \[ P(Q^s) = P_0 \]
   while \( P_0 \) is the price such that each firm has zero profits.

b) \[ Q^d = 1000 - 40P \]
   LR price is determined by the 0-profit condition, i.e., the minimal cost condition, i.e.,
   \[ P^* = LRAC = LRMC \implies \]
   \[ 10Q^2 - 60Q + 100 = 30Q^2 - 120Q + 100 \]
   \[ 60Q = 20Q^2 \implies Q^*_f = 3 \]

Then \[ P^* = 10(9) - 60(3) + 100 = 90 - 180 + 100 = 10 \]

Then \[ Q^*_{market} = 1000 - 40(10) = 1000 - 400 = 600 \]
   \[ N_{firms} = \frac{Q^*_{market}}{Q^*_f} = \frac{600}{3} = 200 \]

c) Now \[ Q^d = 800 - 40P \], \[ Q^*_f = 3 \] & \[ P^* = 10 \] shill.
   \[ Q^*_{market} = 800 - 40(10) = 400 \]
   Then \[ N_{firms} = \frac{400}{3} \approx 133 \]
3) \( p \geq 341 \), \( \Rightarrow \) \( SRTC = Q^3 - 11Q^2 + 100Q + 1000 \)

a) \( FC = 1000 \)

b) \( \text{SRAVC} = \frac{\text{SRVC}}{Q} = \frac{Q^3 - 11Q^2 + 100Q}{Q} = Q^2 - 11Q + 100 \)

c) \( P = 60 \), \( MR = 1 \), set \( MR = MC \)
\[ MC = 3Q^2 - 24Q + 100 \]

\[ 3Q^2 - 24Q + 100 = 60 \Rightarrow 3Q^2 - 24Q + 40 \]

\[ \text{Quadratic formula} \]
\[ \text{when } aX^2 + bX + c = 0, \quad X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ \text{In this case we ignore negative solutions since } Q \text{ must be } \geq 0 . \]

\[ 50 \]
\[ Q = \frac{-24 \pm \sqrt{(24)^2 - 4(3)(40)}}{2(3)} = \frac{-24 \pm \sqrt{496}}{6} \]

\[ = 8 \pm \frac{13.44}{6} = 8 \pm 2.24 \]

\[ 50 \quad Q^* = 8 + 2.24 \approx 9.243 \]

**BUT** in the short run, Hack will only operate if \( AVC < P \) at \( p = 60 \).

In particular, when \( AVC = MC \) we find the minimum price at which Hack will produce.

i.e. \( 3Q^2 - 24Q + 160 = Q^2 - 11Q + 160 \Rightarrow 2Q^2 = 12Q \)

\[ \Rightarrow Q = 6 \Rightarrow AVC(6) = Q^2 - 11Q + 160 = 64\]

Since \( 60 < 64 \), Hack will produce \( Q \) since he can't even recover some of his fixed costs.

d) Yes, Hack should produce since \( Q > 64 \).

At \( p = 60 \), \( P\text{it} = 1000 \), \( PS = 0 \), hence he won't produce.
4) By 343, #15
   a) Since the fee is a fixed cost, it won't affect Iliana's output, but it will decrease her profit.
   b) In the long run, Iliana will exit the industry since she is now operating at a loss.
   c) This requirement will increase marginal variable cost and so decrease Iliana's optimal output in the short run as well as her profits. The same holds in the long run. In the short run, she may continue to produce but in the long run, she will exit the industry.

5) By 393, #5: \( TC = x^3 - 15x^2 + 100x + 30 \)
   a) \( MC = 3x^2 - 30x + 100 \)

\[ \frac{\Delta MC}{\Delta x} = 6x - 30 = 0 \Rightarrow x = \frac{30}{6} = 5 \]

So when \( 0 < x < 5 \) \( MC \) is decreasing

6 \( x > 5 \) \( MC \) is increasing.

b) \( p^* = 52 \), set \( MC = 52 \) so \( 3x^2 - 30x + 100 = 52 \)

So \( 3x^2 - 30x + 48 = 0 \Rightarrow x^2 - 10x + 16 = 0 \)

\( = \) \( x^* = \frac{10 \pm \sqrt{100 - 4(16)}}{2} = \frac{10 \pm 2\sqrt{6}}{2} = 5 \pm 3 \)

So \( x^* = 2 \) or \( 8 \), whichever gets higher profit.

\[ \Pi(x) = 52(2) - [2^3 - 15(2^2) + 100(2) + 30] = 52 - 24 \]

\[ \Pi(8) = 52(8) - [8^3 - 15(8^2) + 100(8) + 30] = 52 - 24 \]

So \( x^* = 8 \), \( \Pi^{*} = 34 \).
6) \( P = 3Q, \quad 3Q = 100 - P, \quad TC = 10Q + 0.5Q^2 \)

a) \( TR = PQ = (100 - Q)Q = 100Q - Q^2 \)

b) \( \Pi = 100Q - Q^2 - 10Q - 0.5Q^2 = 90Q - 1.5Q^2 \)

c) \( \frac{d\Pi}{dQ} = 0 \Rightarrow 90 - 3Q = 0 \Rightarrow Q^* = 30 \)

d) \( P^* = 100 - Q^* = 70, \quad \Pi^* = 70(30) - 10(30) - 0.5(30)^2 = 1,350 \)

e) \( PS^* = (P^* - AVC)Q^* = 30(1070 - 10 - 0.5(30)) = 1350 \)

f) \( ES^* = \int_0^{30} 100 - Q - 10dQ = [300Q - \frac{1}{2}Q^2]_0^{30} = 450 \)

g) If they're in perfect competition, equilibrium is given by \( P(Q^D) = MC(Q^D) \) so

\[
100 - Q = 10 + Q \Rightarrow 90 = 2Q \Rightarrow Q^* = 45
\]

Then \( P^* = 100 - 45 = 55 \)

so \( PS^* = 45(55 - 10 - 0.5(45)) = 1012.50 \)

\[
CS^* = \int_0^{45} 100 - Q - 55dQ = [100Q - \frac{1}{2}Q^2]_0^{45} = 1012.50
\]

Then \( DWL = CS^* + PS^* - CS^m + PS^m \)

\[
= 1012.50 + 1012.50 - 1350 - 450 = 2095 - 1800 = 295
\]

7. a) Google: They have a technological advantage providing better search. They could lose their monopoly by not spending enough money to preserve their technological lead.

b) Facebook: Switching costs - the point of a social network is to network with other people, so no one wants to go to a competitor with less users. They could lose their monopoly if they make their consumers angry enough with ads or a new, cooler social network attracts new users.
c) Local Power Company: Economics of scale to probably government regulation. New solar or wind technology that allows you to generate your own power at home could overturn the monopoly. Also deregulation could do it.

d) Local Water Company: again, economics of scale to government regulation. Deregulation could break the monopoly, or maybe cheap technology for purifying your own water.