Econ 301 Intermediate Micro
Problem Set 2 Solutions

1. \( P(Q) = 600 - 0.01Q^2 \) \( P'(Q) \cdot Q = 100 + 0.04Q^2 \)

   a) \( 600 - 0.01Q^2 = 100 + 0.04Q^2 \)

   \( 500 = 0.05Q^2 \)

   \( Q^2 = \frac{500}{0.05} = 10000 \)

   \( Q = 100 \)

   \( P^* = 600 - 0.01(10000) = 500 \)

   \( b) \quad CS^* = \int_0^{Q^*} P(Q) - P^* \, dQ \)

   \( = \int_0^{100} 600 - 0.01Q^2 - 500 \, dQ \)

   \( = \left[ 100Q - \frac{0.01}{3}Q^3 \right]_0^{100} \)

   \( = 100(100) - \frac{0.01}{3}(100)^3 - \left[ 100(0) - \frac{0.01}{3}(0)^3 \right] \)

   \( = 10,000 - \frac{10,000}{3} = 6,666.67 \)
\[PS^* = \int_0^{Q^*} p^* - p(Q^*) \, dQ = \int_0^{100} 500 - 100 - 0.04Q^2 \, dQ\]

\[= \left[100Q - \frac{0.04}{3}Q^3\right]_0^{100} = \$1,264.67\]

c) \[P_{tax}(Q^D) = P(Q^D) - T = 600 - 0.01Q^2 - 2\]

\[= 598 - 0.01Q^2\]

so new equilibrium:

\[598 - 0.01Q^2 = 100 + 0.04Q^2\]

\[498 = 0.05Q^2\]

\[Q^2 = \frac{498}{0.05} = 9960\]

\[Q_{tax} = \sqrt{9960} = \$99.80\]

\[P_{tax} = 598 - 0.01(9960) = \$498.40\]

\[P_{tax} = \$500.40\]

Note: I set the problem up so that the tax is on the consumers. All answers on the producers are the same.
d) \( CS^{tan} = \int_0^{Q_{tan}} (P(Q) - P_{tan}) \, dQ = \int_0^{99.8} (600 - 0.01Q^2 - 500) \, dQ \)

\[ = \left[ 600Q - \frac{0.01}{3}Q^3 \right]_0^{99.8} = 6,626.71 \]

\( PS^{tan} = \int_0^{Q_{tan}} (P(Q) - P_{tan}) \, dQ = \int_0^{99.8} (198.4Q - 100 - 0.04Q^2) \, dQ \)

\[ = \left[ 99.8Q^2 - \frac{0.04}{3}Q^3 \right]_0^{99.8} = 2,6506.83 \]

\( TR^{tan} = T^\circ Q_{tan} = 99.8(\pi) = 199.60 \)

e) \( DWL^{tan} = CS^x + PS^x - CS^{tan} - PS^{tan} - TR^{tan} \)

\[ = 6,666.67 + 2,666.67 - 6,626.71 - 2,6506.83 - 199.60 = 99.20 \]
2. Quota at 30 purses

a) \[ p_{\text{quota}} = \mathbb{P}(Q^0 = 30) = \frac{600 - 0.01(30)^2}{591} \]

\[ = \$591 \]

\[ \text{so} \quad CS_{\text{quota}} = \int_0^{30} p_{\text{quota}} \, dQ = \int_0^{30} 600 - 0.01Q^2 - 591 \, dQ \]

\[ = \left[ 9Q - 0.01Q^3 \right]_0^{30} = 180 \]

\[ PS_{\text{quota}} = \int_0^{30} p_{\text{quota}} - \mathbb{P}(Q^0) \, dQ = \int_0^{30} 591 - 100 - 0.04Q^3 \, dQ \]

\[ = \left[ 491Q - 0.04Q^3 \right]_0^{30} = 14,370 \]

b) \[ DWL_{\text{quota}} = CS^k + PS^k - CS_{\text{quota}} - PS_{\text{quota}} \]

\[ = 6,666,667 + 2,300,000 - 180 - 14,370 \]

\[ = 8,188,036.4 \]

c) \[ p_{\text{floor}} = \$591, \text{so} \quad Q_{\text{floor}} = \frac{600 - 0.01Q^2}{591} \]

\[ 591 = \mathbb{P}(Q^0) = 600 - 0.01Q^2 \]

\[ Q^2 = \frac{9}{0.01} = 900 \]

\[ Q = 30 \]
So \( CS_{\text{floor}} = \int_0^{Q_{\text{floor}}} P(Q) - p_{\text{floor}} dQ = \int_0^{30} 600 - 0.01Q^2 - 8Q dQ \)

\[ = \left[ 9Q - \frac{0.01}{3}Q^3 \right]_0^{30} = 180 \]

\( PS_{\text{floor}} = \int_0^{Q_{\text{floor}}} P(Q) - P(Q^*) dQ = \int_0^{30} 591 - 100 - 0.04Q^2 dQ \)

\[ = \left[ 591Q - \frac{0.04}{3}Q^3 \right]_0^{30} = 14,370 \]

Same as 4) \( \text{The quota!} \)

d) \( DwC_{\text{floor}} = CS^* + PS^* - CS_{\text{floor}} - PS_{\text{floor}} = 18,783.34 \)

Same as the quota!

e) elf a quota at \( Q \) induces a market price at \( P \) if a price floor at \( P \) induces an equilibrium quantity of \( Q \), then there is no difference between the two in terms of \( CS, PS, \) etc.

This is true in general — every price regulation has a corresponding quantity regulation that accomplishes the same thing. The caveat of course is that we are ignoring costs of implementation.
3. I made a typo which made this problem very difficult to solve by hand.

The correct inverse demand curve is

\[ P(Q^d) = 10 - 2Q^d \text{ not} \]

\[ P(Q^d) = 10 - 2Q^2. \]

So this question was not graded.