Econ 301 Intermediate Microeconomics
Chapter 3 In Class Exercise 2

Answer all questions on a separate sheet of paper. Make sure you show your work.

1. Consider again the market for large artisanal chocolate chip cookies in cookietown. Suppose this market has the following inverse supply and demand curves:

\[ P(Q^D) = 10 - 0.1 \times Q^{1.5} \]
\[ P(Q^S) = 1 + 0.03 \times Q^{1.5}. \]

Price is in dollars per cookie and quantity is in thousands of cookies. Remember from the last exercise, you’ve already found the equilibrium price and quantity and the consumer and producer surpluses under the unregulated market equilibrium.

(a) Suppose the chamber of commerce in cookietown decides to collect a sales tax of $1 per cookie from artisanal cookie producers in order to fund a new gymnasium for the cookietown high school. Write down the new inverse supply curve and compute the new equilibrium quantity, the price that the consumers pay, and the price that the producers receive.

(b) Using the information from the last part, compute the consumer surplus, producer surplus, government revenue, and deadweight loss under the tax.

(c) Suppose instead that the tax is collected from cookie consumers. Redo parts a) and b) under this scenario.

(d) Suppose instead of taxing the producers, the chamber of commerce decides to subsidize the cookie market by paying them $1 for each cookie they sell. Write down the new inverse supply curve and compute the new equilibrium quantity, the price that the consumers pay, and the price that the producers receive.

(e) Using the information from the last part, compute the consumer surplus, producer surplus, government revenue, and deadweight loss under the subsidy.
Again \( P(Q^0) = 10 - 0.1Q^{1.5} \), \( P(Q^s) = 1 + 0.03Q^{1.5} \) | ch3 ex 2

Solution

From the last exercise we have the equilibrium unregulated quantities:
\[ p^n = 3.08, q^r = 16.86, c^s = 69.99, p^s = 210.6 \]

a) Tax on producers of $1

\[ s_t = p^s(q^s) = p(q^s) \times t \]
\[ t = \text{tax} = $1 \]

If you want producers to produce a certain quantity, you have to pay them more – exactly the tax more.

Now to find \( q^t \):

\[ p_t(q^s) = p(q^0) \Rightarrow 1 + 0.03Q^{1.5} = 10 - 0.1Q^{1.5} \]
\[ 0.13Q^{1.5} = 8 \Rightarrow Q^t = \left( \frac{8}{0.13} \right)^{\frac{1}{1.5}} = 15.59 \]

Then
\[ p^d_t = 10 - 0.1(15.59)^{1.5} = $3.84 \]
\[ p^s_t = p^d_t - t = $2.84 \]

b) \( LR = Q^t(1) = 15.59(1) = $15.59 \)

\[ CS = \int_0^{Q^t} (P(Q^d) - p^d_t)dQ = \int_0^{15.59} (10 - 0.1Q^{1.5} - 3.84)dQ \]
\[ = \left[ 6.16Q - \frac{0.1}{2.5}Q^{2.5} \right]^{15.59}_0 = $56.65 \]
\[ PS = \int_0^{q^T} P_T^s - P(Q^s) \, dQ = \int_0^{15.59} 2.84 - 0.03Q^{1.5} \, dQ \]
\[ = \left[ 1.84Q - \frac{0.03}{1.5}Q^{2.5} \right]_0^{15.59} = 17.17 \]

Then, \[ DWL = CS^* + PS^* - PS - CS - CR \]
\[ \text{from previous exercise} \]
\[ = 69.99 + 6.06 - 17.17 - 56.65 - 15.59 \]
\[ = 40.64 \]

C) Now the tax is on consumers

\[ P_{\text{ex}}(Q^D) = P(Q^D) - T \]

in order to get consumers to purchase a given quantity of the good, you have to charge them \( T \) less when you tax them by \( T \).

So, \[ P_T(Q^D) = 10 - 0.1Q^{1.5} - 1 = 9 - 0.1Q^{1.5} \]
so, set \( P_T(Q^D) = P(Q^D) \Rightarrow 9 - 0.1Q^{1.5} = 1 + 0.03Q^{1.5} \)
then \[ 8 = 0.13Q^{1.5} \Rightarrow Q^T = \left(\frac{8}{0.13}\right)^{1/1.5} = 15.59 \]

same \( Q^T \) as when the tax is imposed on the producers!

So, \[ P_T^D = P(Q^D) \] is the same (plugging in \( Q^D = Q^T \))

\[ P_T^S = P(T) \] is also the same

So nothing changes! - It doesn't matter whether the tax is on consumers or producers once the taxes are the same.
d) A subsidy is just a negative tax, but otherwise everything else for the market is the same.

The graph, however, is convoluted, so it's last:

So \( P_{\text{tax}}(Q^s) = P(Q^s) + T = 1 + 0.03Q^{1.5} + (-1) = 0.03Q^{1.5} \)

while \( P(Q^d) = 10 - 0.1Q^{1.5} \) (stays the same)

So set \( P_{\text{tax}}(Q^s) = P(Q^d) = \)

\[ 0.03Q^{1.5} = 10 - 0.1Q^{1.5} \Rightarrow Q^{1.5} = \frac{10}{0.13} \]

\[ \Rightarrow Q_T = \left( \frac{10}{0.13} \right)^{1.5} = 18.09 \]

Then \( P_T = P(Q^d) = 10 - 0.1(18.09)^{1.5} = 2.31 \)

\( P_T = P_T^d - T = 2.31 - (-1) = 3.31 \)

e) \( CS = \int Q_T P(Q) - P_T^d dQ = \int_{18.09}^{18.09} [10 - 0.1Q^{1.5} - 2.31] dQ \)

\[ = \int_{0}^{18.09} 7.69 - 0.1Q^{1.5} dQ = \left[ 7.69Q - \frac{0.1Q^{2.5}}{2.5} \right]_{0}^{18.09} \]

\[ = 83.49 \]

\( PS = \int Q_T P_T^{s} - P(Q^s) dQ = \int_{18.09}^{18.09} [3.31 - 1 - 0.03Q^{1.5}] dQ \)

\[ = \left[ 2.31 - 0.03Q^{1.5} \right]_{0}^{18.09} = 2.5\times 0.9 \]
\[ GR = Q^T \cdot T = 18.09(-1) = -18.09 \]

\[ DWL = CS^* + PS^* - CS^* - PS^* - GR \]

\[
from \ last \ time \\
= 69.99 + 21.06 - 83.44 - 25.09 - (-18.09) \\
= -0.61
\]

The graph:

\[ CS = A + B + D + E \]

\[ PS = G + D + B + C \]

\[ GR = -B - C - D - E - F \quad (\text{Revenue is negative}) \]

\[ DWL = F \]

Consumer’s share of the subsidy: \( D + E \)

Producer’s share of the subsidy: \( B + C \)

The reason the graph is convoluted is because \( CS + PS \) overlap each other and the \( GR \).